The Wave Theory of Numbers

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Abstract

The concept of waves is very fundamental to classical and modern physics alike, being essential in describing light, sound, and elementary particles, among many other phenomena. In this paper, we show that the wave/particle duality is a phenomenon manifested not only in the physical world and the mathematics that describes it, but also in the simple numbers that form the basic matrix upon which most of our sciences rest. We will also show how this wave-based approach to numbers could be essential to our understanding of the mathematical and physical constants that govern the physical laws as well as the natural elements emerging from them.

I. INTRODUCTION

Waves are one of the most studied natural phenomena and in most scientific fields. This is due to their ubiquitous presence everywhere in the world around us, from the macrocosmic level to the microcosmic one.

The most common form of waves is the one observed on water surfaces, mostly due to the mechanical forces that winds exert on these surfaces like the seas, lakes, ponds, etc. Or simply drop a stone into still water and you will generate circular waves that expand outwardly from the center (where the stone was dropped).

Sound also propagates in the form of waves that travel by compression and expansion of air molecules[1].

When waves emanating from two, or more, sources meet, they interfere with each other creating a pattern of alternating light and dark regions, being the result of the constructive and destructive interference between the troughs and crests of the waves. So, when troughs or crests meet, they add up constructively, and when a trough and crest meet, they add up destructively, in other words, cancel each other.

This pattern is one of the main proofs on the wave-like nature of light, as demonstrated by the famous Young's double-slit experiment[2].

In fact, interference patterns are the hallmark of waves propagation and one of the most important tools used by scientists allowing them to measure important qualities such as the frequencies of the interfered waves, the speeds of the sources emitting them, the distances of these sources, among many other properties of interest.

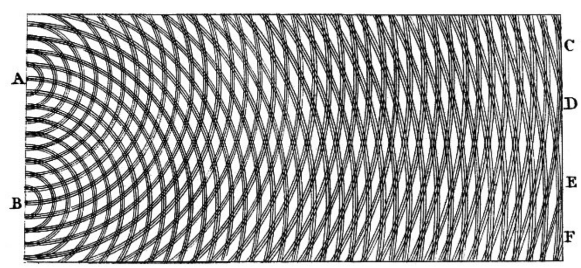


Fig.1: A sketch by Thomas Young illustrating light diffraction-interference pattern.

At the beginning of the 20th century, it became evident that the classical mathematical models used in physics for centuries are not able anymore to explain the new observations and discoveries the experimental physicists were making.

The breakthrough came in 1913 when Niels Bohr, a Danish physicist, suggested a novel way to look at the orbit of the electron around the proton in the hydrogen atom; instead of a continuum of possible energy states, only discrete or quantized ones are allowed[3]. The success of his model in predicting the spectrum emitted by the hydrogen

atom was the spark that ignited the quantum mechanics revolution.
Another breakthrough came when the French physicist Louis De Broglie suggested that particles may also behave like waves inside the atoms; having wavelengths and frequencies just like sound and light do[4].
This theory explained many observations and opened the door into an exciting new reality that is, however, not easy to comprehend nor to visualize[5].
The wave-particle hypothesis was confirmed in 1927 when another kind of Young's double-slit experiment had been performed, the so-called Davisson-Germer[6] experiment, where a stream of electrons replaced that of light and where interference diffraction patterns were also observed.
On the other hand, earlier on in 1905, Albert Einstein suggested in his photo-electric paper[7] that light is made of discrete wave packets of energy called photons. Thus, light as well behaves either like a wave or as a point-like entity, depending on the observation or the experiment performed.
Consequently, the term *particle/wave duality* was coined to describe this strange phenomenon.

Even though this duality has fascinated people ever since, nowadays, however, it has been taken for granted; a bizarre quality of nature that we have to contend with in order to continue our scientific advancement.
But where does this duality come from? And to what level do its ramifications reach? We definitely can't feel it in our macrocosmic level. Maybe we need to go to smaller levels to detect its origin, smaller even than the fundamental particles themselves.
There are many levels below that of the elementary particles. For example, protons and neutrons are believed to be made of even smaller entities called quarks, three of them in each[8]. And in String Theory[9], everything, including quarks, is supposed to be made of tiny vibrating strings, which could be the most fundamental level physicist have theorized.

But there could be still a more fundamental level; that of the physical and mathematical constants which govern space and natural forces, such as Pi, Euler number, alpha, etc.
Could these constants, and the numbers that make them, hold the key to the duality enigma?
This we explore in this paper.

II. THE PARTICLE-ASPECT OF NUMBERS

The idea behind atoms goes way back to the 5th century BC, where matter was thought by the Greek philosopher Leucippus[10] to be continuously devisable until one reaches a level where it cannot be divided anymore; that is the level of atoms.
But at the beginning of the twentieth century, atoms were found to be made of even smaller entities, called elementary particles, mainly the electron, the proton, and the neutron.
Consequently, the field of quantum mechanics emerged in order for physicists to have a mathematical framework able to describe these particles and their interactions.
One method to do so is by assigning specific numbers to these particles, called *Quantum Numbers*[11], that describe their quantum states. These quantum numbers are conserved and dictate how the particles behave and whether a certain reaction is allowed or not.
For example, in the beta-decay, where a neutron decay into a proton, an electron, and an antineutrino, the quantum charge number should be conserved before and after the interaction:

$$n^0 \rightarrow p^+ + e^- + \bar{\upsilon} .$$

And we see that 0 = 1 – 1 + 0, as the proton is considered positive, the electron negative and the antineutrino carries no charge just like the neutron.

There are many other qualities assigned to particles, such as angular momentum, principle quantum number, spin, etc. and most of them are described by sets of integers either discrete similar to [-1, 0, 1], as in the charge number for example, or continuous like in the principle quantum number [1, 2, 3...]

But we are not constrained to use these specific numbers only. We can use numbers from 1 to 9 to describe the elementary particles and their behaviors (this, however, require the application of the digital root math[12].)

For example, if we to assign particles the numbers shown in the table below, we find that these numbers also satisfy the beta-decay, and any elementary reaction, as follows: 1 = 2 + 3 +5 = 10 and the digital root of 10 = 1.

PARTICLE	ANTI-PARTICLE
$\boldsymbol{n^0} \rightarrow \mathbf{1}$	$\overline{n^0} \rightarrow 8$
$\boldsymbol{p^+} \rightarrow \mathbf{2}$	$p^- \rightarrow 7$
$\boldsymbol{e^-} \rightarrow \mathbf{3}$	$e^+ \rightarrow 6$
$\boldsymbol{v} \rightarrow \mathbf{2}$	$\overline{v} \rightarrow 2$

Notice that the particles are assigned these numbers such that when each particle is added to its antiparticle, the result will be number 9, which is equivalent to 0 in the digital root math. However, number 9 is not 0 or nothingness, and this where the two systems differ; as when a particle and antiparticle unite, they annihilate each other of course; however, they produce a great amount of energy in the process. And this is exactly what number 9 is telling us here: that the unification results in energy, not 0 as numbers 1 and -1 will produce.

Therefore, numbers can incarnate the role of particles perfectly (and energy, as in number 9). They even dictate how particles interact and behave.

In a sense, it is from the properties of numbers that the physical reality inherits its fundamental laws.

III. THE WAVE ASPECT OF NUMBERS

We have seen how, in their point-like state, particles and numbers can be interchanged; being different manifestations of the same aspects.

However, as we have shown above, this point-like aspect is not the only one that particles exhibit; they have a wave-like nature that is as fundamental as the point-like one. This wavy nature is described by wave equations, such as Schrodinger's equation[11], where the wave is interpreted as a probability density spread over time and space; only when we square it that its physical meaning can be understood.

So, could numbers have a wave-like nature also? In other words, does the wave-particle duality apply to numbers as well?

There are two main types of numbers: integers and non-integers (floats).

Integer numbers are exact, with no commas or leftovers. So, an integer would be something like 1, 2, 3, 564, 9845603 etc.

A non-integer, or float, on the other hand, would be something like 5.65, or 2.7182818…

For the case of floats, there are many types. There are those that are rational, having few numbers for their remainders or where the remainder repeats indefinitely in a specific pattern, like 1/7 = 0.142857142857…

And there are irrational numbers, being those numbers where their remainders repeat in a completely random fashion, but still are the solution of some algebraic equation, such as the golden section Phi[13].

On the far extreme, we have transcendental numbers, with their remainders repeat indefinitely in a string of random digits. Unlike transcendental numbers, however, they are not the solution to any known algebraic equation, such as π or Euler number *e*.

The ratio 1/7 is particularly interesting as it seems to capture the essence of the wave. This is because the repeated pattern of its remainder [142857] mimics a wave when plotted individually, and more so collectively, as shown in figure (2) below.

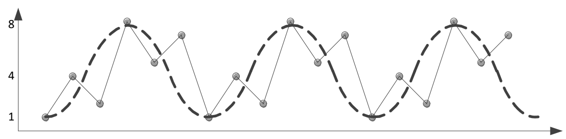


Fig.2: The wave-like nature of the remainder of 1/7.

In fact, taking the inverse of numbers is the simplest mathematical way to create repeated patterns, for example, 1/6 = 0.16666…, or 1/273 = 0.00366300366300… etc.

On the other hand, integer numbers [0, 1, 2, … ∞] only increase in magnitude and, consequently, no wave-like behavior is observed. However, when looking at their digital roots, we find a wave-pattern embedded underneath.

So, when numbers from 1 to infinity are distributed, let say within three columns, we find that the digital roots of the numbers in each column repeats indefinitely, as shown in the table below.

NATURAL NUMBERS			DIGITAL ROOT		
1	2	3	1	2	3
4	5	6	4	5	6
7	8	9	7	8	9
10	11	12	1	2	3
13	14	15	4	5	6
16	17	18	7	8	9
…	…	…	…	…	…

The 1st column's digital root repeats in the sequence [1, 4, 7]. This sequence resembles a wave, just like 1/7 does. And so is for the other two columns made of [2, 5, 8] and [3, 6, 9].

Therefore, in principle, numbers do observe some form of a numerical wave-like pattern. We can actually say that numbers carry certain frequencies, either individually as in their repeating remainders, or collectively as in their digital roots.

But, if this wave-like essence is related to the same one we observe everywhere in nature, it should also exhibit some of its properties. And there is no property of waves more fundamental than interference.

IV. NUMBERS' INTERFERENCE AND PATTERNS

Of the most basic interference patterns, is the one generated from two point-like sources producing propagating circular waves. The emerging pattern is similar to the one produced by the two-slit experiment, shown in Figure (1) and reproduced in more detail below.

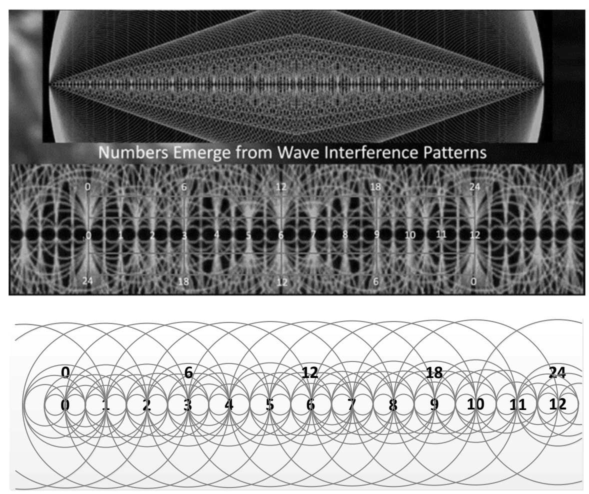


Fig.3: (top) Interference pattern from two point-like sources. (Middle) The same interference pattern magnified to illustrate the 12-based fractal nature of the pattern. (Bottom) Digital rendering of the interference pattern for a scale from 0 to 12.

By close inspection of the above pattern, we notice that the waves interference creates periodic self-similarities or fractals with a frequency value of multiples of 12, depending on what level of the fractals you are at.

Moreover, the pattern is built on doubling principles where each circle encompasses an even number of smaller circles inside it and so on.

Let us examine the interference pattern on a bigger scale as shown in figure (4) below (a larger rendering of this image is in Appendix A along with more detailed explanation).
This numerical wave pattern is scalable in a sense that we can apply it to numbers from 1 to 10 as well as from to 100, or to 2000, etc.
For our case, we look at a range from 1 to 121 for reasons explained shortly. The numbers on the horizontal axis are referenced by their digital root instead of their original values as to make the pattern less complicated.

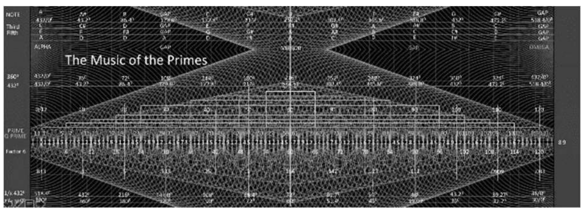


Fig.4: An illustration of wave-interference pattern from two-point-like sources and for a scale between 0 and 121, along with prime and Quasi-prime symmetry, angular references, and musical notes correspondences. (Please refer to Appendix A for a more detailed explanation.)

Notice how the pattern is symmetric around number 60, especially when considering prime numbers.
Prime numbers are unique because they are divisible by themselves and number 1 only. The Fundamental Theorem[14] of Arithmetic states that any integer >1 can be expressed as the unique product of two or more primes. In this sense, prime numbers can be considered the main block upon which all other numbers are built, giving them a special status among the rest.

So, starting with number 60, we notice that the two numbers surrounding it, 59 and 61, are both prime numbers.
Going further to numbers 54 and 66, which are symmetric around 60, we find another symmetry emerging with 53 and 67 being primes and 55 and 65 are not. (We label these non-prime numbers as Quasi Primes, being the product of prime numbers that are ≥ 5, thus excluding 2 and 3. Please refer to reference 15 for more information on these numbers[15].)

Numbers 47 and 49 on the left skips their matching pair and match up instead with the next pair of 77 and 79, while 71 and 73, both prime, match up with 41 and 43, also primes. These four pairs form an offset that creates a kind of momentum around number 60.
Further out, we go back to the main symmetry with pairs 35 and 37 matching up with 83 and 85.

When we reach numbers 30 on the left and 90 on the right, we lose our reflection symmetry for the pairs [29, 31] and [89, 91]. This is because, at 90, we have reached the fractal limit of our 9-base system, starting with 9, then 90, the 900, etc. Therefore, the break in symmetry here indicates the end of one cycle and the beginning of another. (We are working in a 9-based system not a 10-based one because 10 is reducible to 1 in the digital root math, and hence we have 9 numbers only, from 1 to 9.)
One other cycle is at number 24, where the twin primes consistency around numbers that are multiples of 6 breaks and the first quasi-prime number appears, being number 25.
At 114 we get another break of symmetry, paving the way for the appearance of the first quasi-prime pair of [119, 121] and with the cycle finishing at 120, as in a full octave.

Thus, matching numbers on the waves' interference pattern helped us identify some interesting observation, such as the 12-based fractal system, the symmetrical relationships around number 60, especially between prime and quasi-prime numbers.

With the length of the wave pattern being a fractal of 12, thus, it matches the western 12-based musical octave. We can then superimpose the notes of the octave on the pattern as if the

interference is generated by sound waves, as shown in figure (4) above.
We used the notes of the major 5th octave with the note A_5 tuned to the standard pitch of 432Hz. This is because by using this tuning value, we get a perfect correspondence between the angular reference of the notes' and the internal angles of specific polygons as explained in detail in reference[16] 16.

Therefore, the wave interference pattern is a matrix where numbers, as well as geometry and sound, can all be manifested and expressed, which in return allows us to find new connections between these various disciplines, based on the fractal level we are working at. This will have a profound implication on our understanding of the universe, how it works and how everything is related even when there are no apparent relationships, like between elements and music as we to discover later on.

V. WAVE INTERFERENCE AND THE EMERGENCE OF FUNDAMENTAL CONSTANTS

The geometry the interference pattern creates is essentially a numbering or a measuring system, which sets the scale that determines the properties of the physical qualities that depends on these waves.
For example, if we to consider these waves as electro-magnetic ones emanating from the fluctuation of the zero-point energy[17], then their interference pattern would set the scale that controls many physical laws and qualities. So, in the vacuum, the waves set a scale that determines qualities such as the speed of light. Consequently, in a non-vacuum medium, the scale will change depending on the structure of the medium and the speed of light will change accordingly.

We can also think of each point on the interference pattern as an individual number working as a point-like source and creating its own propagating numerical waves. This logic is synonymous to Huygens–Fresnel Principle[18] where each point on the wavefront is treated as an individual point-like (or dipole) source of waves, which is very powerful in explaining many properties of light that are not explainable using the single source model.
Thus, mathematical constants such as π or e can be thought of emerging from specific integers plus a wave. For example, the constant π = 3.14592… is nothing but number 3, representing the particle aspect of the number or its amplitude, plus a remainder of 0.141592…
To make the concept more comprehensible, let us take π as the ratio of 22/7 = 3.142857… This is equivalent to 3 + 1/7 where number 3 represents the amplitude of the π-number while the ratio of 1/7 represents its phase.
The same logic is applicable to Euler number e = 2.718… which can be expressed as 19/7 = 2.714285… = 2 + 5/7, with 2 being the amplitude and 5/7 being the phase.

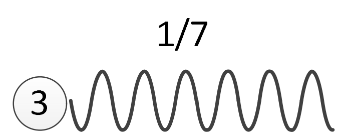


Fig.5: Number π expressed in a wave/particle fashion; the sum of an integer amplitude (3) and a phase (1/7).

Thus, each number on the wave pattern form a seed or a kernel around which mathematical and physical constants grow through the addition of a phase that represents the remainder of the constant.
Form this knowledge, and by deducing the mechanism by which the phases are generated, we can start predicting the values of other constants that are not yet identified. This is being investigated thoroughly at the moment and will be discussed in the next paper.

We can also take a more geometrical approach to the mathematical and physical constants as being *natural separations of the waves*.

For example, number π can be thought of as emanating from the wave-separation of unity in the ratios of 3 and 4 as illustrated in the image below to the left. The rest of the remainder can be generated from the same process; endless doubling or separation of the circular waves. And so is the case for the rest of the mathematical constants; emerging from the overlapping or separation of propagating waves.

This is a powerful representation that morphs the abstract numerical constants into more comprehensible aspects, with each constant having its own unique wave-signature, which could very well be how these constants are communicated to the universe and also the mechanism by which it governs its physical laws

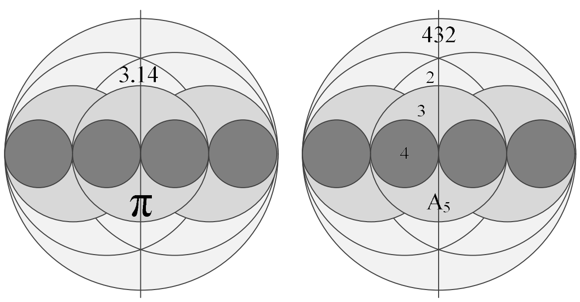


Fig.6: Mathematical constants represented as doubling of waves/circles; for π (left), and for the standard tuning of 432Hz (right).

VI. THE PERIODIC WAVE OF ELEMENTS

The natural elements that make up our universe are usually listed in a linear fashion in what is called the Periodic Table of Elements, first introduced by the Russian chemist Dmitri Mendeleev in 1869. In this table, the elements are distributed in 14-18 columns or families based on their atomic number (the number of protons in the nucleus) and their chemical properties.

Using the wave theory approach, we can separate the elements in a more natural way that depends on the wave interference pattern instead of the linear method used in the regular periodic table.

We start with the first unity wave/circle that encompasses all the elements, representing the first octave, where we pin hydrogen at the top center; being the simplest comprehensible element, as shown in figure (7) below. (For better visualization, please refer to Appendix B.)

Hydrogen initiates the first separation of the wave into two smaller ones. The next separation generates 4 smaller circles and then 8 of them. These 8 circles can be seen as complementary sine/cosine waves superimposed on top of each other, with one wave corresponding to the next line of elements, from He to Ne, while the other wave corresponding elements from Ne to Ar. (The same elements appear on both sides of the wave because it is a fractal configuration; when one cycle finishes, an exactly similar one starts.)

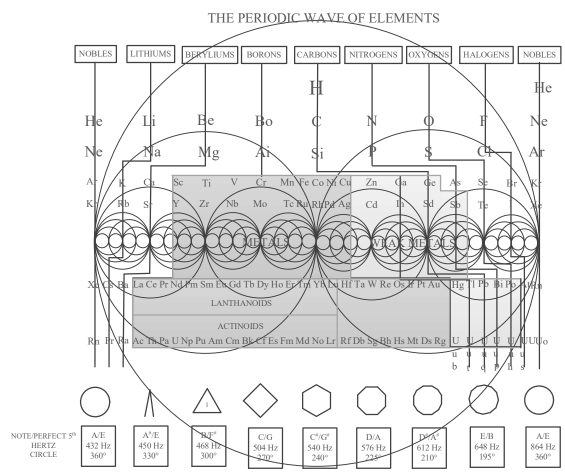


Fig.7: The wave-based table of elements along with their respective families (shaded areas) and their musical notes/geometry correspondences. (Please refer to Appendix B for a more detailed explanation.)

By continuing this octave doubling we find that elements are lined up perfectly in a manner that reflects their respective families, such as Metals, Lanthanoids, Acteonids, etc.

So, noble gases position themselves on both sides of the larger wave or octave, being the chemically inert members of the elements. The more we move closer toward the inside or the peak of the wave the more active and isotopic the elements become.

Notice how each element is positioned at a specific point or node that is determined by the interference of two waves. These waves emanate from the doubling of the previous one that represents the preceding element, which in turns initiates the doubling and the emergence of the next octave elements, and so on. So, for the carbon family, we start with C, then down to Si, Ge, Sn, and finally Pb.

As we showed earlier, the wave pattern corresponds to the musical notes of the octave, which, in turn, corresponds to geometrical shapes. Therefore, we can associate each element with a specific frequency and shape. So, for example, carbon corresponds to note $C_{\#}$ of 540Hz as well as to the hexagon, which is a very appropriate shape as carbon chains usually take the form of hexagons in organic materials. And so is for the rest of the elements.

Another consequence of the above wave-doubling configuration of the elements is the counter-intuitive conclusion that there seem to be missing elements on the hydrogen level, such that when doubled, will generate the next octave of the Carbon level (and also that there could be elements above hydrogen itself).

In fact, if we to follow the wave separation logic to the limit, we need to add 19 extra elements to the already known 118 of them: 3 noble gases, 8 standard elements, and 8 isotopes. This will increase the number of elements to 137, the same number that Richard Feynman, the famous American physicist, had already suggested[19].

This is definitely not an easy reasoning to digest, as what element could be there, simpler than a single electron and a proton? One answer could be that these elements are formed from anti-matter instead of regular matter.

Nevertheless, and whatever the structure of these suggested new elements may be, if any, still the wave-based configuration opens the doors for a novel perspective on the elements and their families that may lead to a better understanding of their quintessence and qualities.

IV. CONCLUSION

By extending the wave/particle duality into the numerical domain, we created a powerful matrix that combines the power of wave interference with the logic of numbers.

The numeric wave interference representation revealed hidden symmetries between numbers, especially prime ones. Treating numbers as natural wave generators, as well as separators, enabled us to develop a different perspective to mathematical and physical constants and how they are generated and manifested, which opens the door for the discovery of many new ones.

Finally, the wave representation was used to rearrange the elements of the periodic table in a more natural way that is based on wave separation, allowing us to gain a deeper understanding of the elements and their respective families, and how they may be related to sound frequencies and geometrical configurations.

APPENDIX

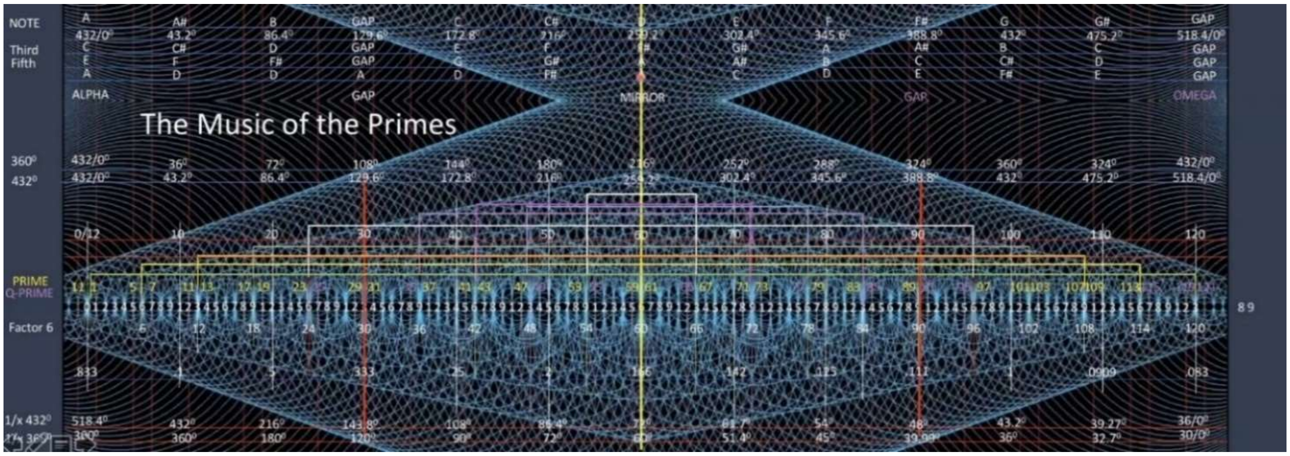
A- Larger rendering of figure (4) with an explanation.

On the main horizontal axis of the wave interference pattern, we find the digital roots of numbers from 1 to 121. On top of them, listed the primes and Quasi-primes positioned in a symmetrical pattern around 60 as explained above. (This number can also be 6, 0.6 or 600, etc. as the whole pattern is of fractal nature.) Above the prime numbers level, we have numbers' angular references as a fraction of 360° and 432°. So, above 60 we have $0.6\times360^{\circ} = 216^{\circ}$ and 288° is just $0.8\times360^{\circ}$ and so on. We use the decimal fraction of the numbers as to maintain the degree references within the 360° circle.

Notice how the cycle ends at 120, corresponding to 432°, which is the same value for the Pythagorean tuning.

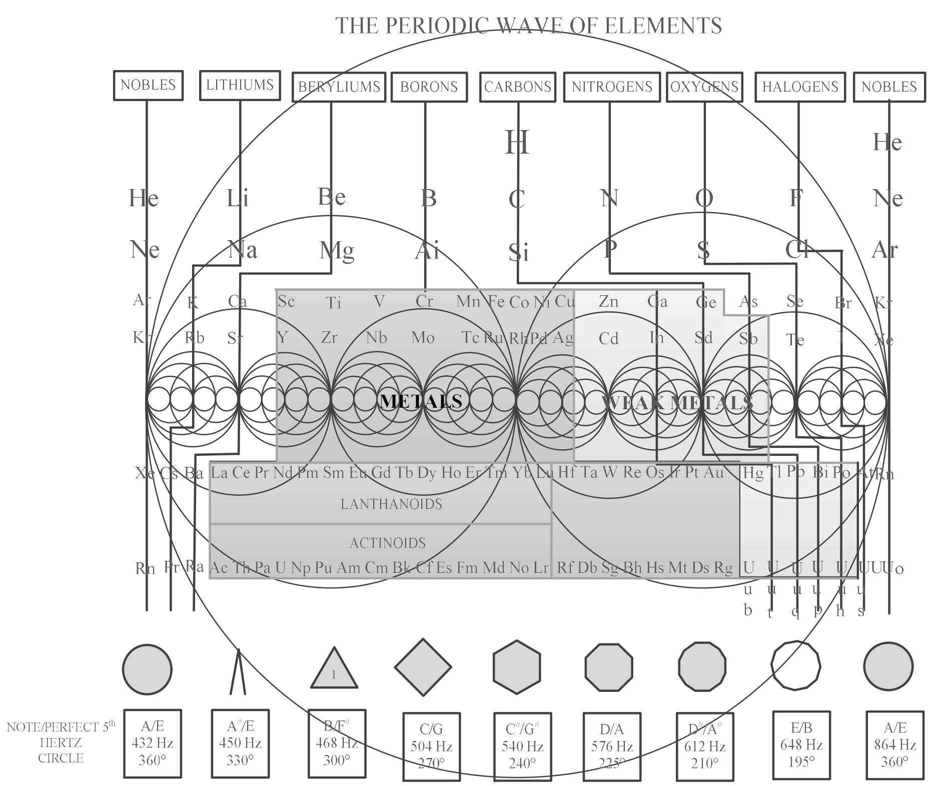
The axis below the digital root numbers is the 1/x reference of the numbers along with their degree references. So, $1/6 = 0.166\ldots$ and $0.166\times432^{\circ} = 72^{\circ}$, and so is the case for the rest of the numbers. The 1/x analysis will be essential when we discuss the constants emergence from the wave theory in the next paper.

The top rows are for the corresponding 12 musical notes and their degrees of reference.



B- Larger rendering of Figure (7) with an explanation.

The wave-based periodic table of elements rests on the wave-doubling principle, with each element initializing the doubling of the wave, which in returns, produces more elements that initiate more doubling and so on. Each node corresponds to specific elements along with a specific musical note and corresponding geometry, as shown at the bottom of the image. The shaded areas represent the chemical families of the elements, such as metals, weak metal, lanthanoids, and actinoids.



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